A thin-wall, double-pipe, counter flow heat exchanger is used to cool oil from T_{in}^{oil} to T_{out}^{oil}. The volume flow rate ($\dot{V}_{oil}$), density (ρ_{oil}), and heat capacity of the oil (c_p^{oil}) are known. Ice water of a known volume flow rate ($\dot{V}_{h_2o}$) is used to cool the oil in the heat exchanger, where the output is actually a 100% saturated steam at (100°C, 1 atm). What is the efficiency of the heat exchanger in equation form?

$$\eta = \frac{E_{OUT}}{E_{IN}} = \frac{|\Delta \dot{E}_{oil}^{fluid}|}{|\Delta \dot{E}_{water}^{fluid}|} = \frac{|\dot{E}_1^{oil} - \dot{E}_2^{oil}|}{|\dot{E}_1^{h_2o} - \dot{E}_2^{h_2o}|}$$

1 – IN , 2 – OUT

$$\rho = \frac{m}{V} \Rightarrow \dot{m} = \rho \dot{V}$$

Energy balance on oil

$$(\dot{Q}_1^{oil} + \dot{W}_1^{oil} + \dot{E}_1^{oil}) - (\dot{Q}_2^{oil} + \dot{W}_2^{oil} + \dot{E}_2^{oil}) = \frac{d}{dt}(\Delta u)_{oil}$$

steady state

$$\Rightarrow \dot{Q}_2^{oil} = \dot{E}_1^{oil} - \dot{E}_2^{oil}$$

where

$$\dot{E}_1^{oil} = \dot{m}_{oil}\left(h_1 + g z_1 + \frac{\vec{V}_1^2}{2}\right)$$
$$\dot{E}_2^{oil} = \dot{m}_{oil}\left(h_2 + g z_2 + \frac{\vec{V}_2^2}{2}\right)$$

$h = u + Pv$

$$z_1 \cong z_2, \quad \vec{V}_1 \cong \vec{V}_2 \Rightarrow \dot{E}_1^{oil} - \dot{E}_2^{oil} = \dot{m}_{oil}(h_1 - h_2)$$

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p \Rightarrow \Delta h = C_p \Delta T$$

$$\Rightarrow \dot{E}_1^{oil} - \dot{E}_2^{oil} = \dot{m}_{oil} C_p^{oil} (T_1^{oil} - T_2^{oil})$$

Δh

$$T_1^{oil} > T_2^{oil}$$

Energy balance on water (ice → steam)

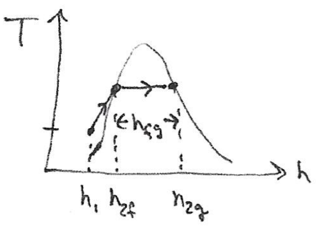
$$(\dot{Q}_1^{h_2o} + \dot{W}_1^{h_2o} + \dot{E}_1^{h_2o}) - (\dot{Q}_2^{h_2o} + \dot{W}_2^{h_2o} + \dot{E}_2^{h_2o}) = \frac{d}{dt}(\Delta u)_{h_2o}$$

steady state

$$\Rightarrow \dot{Q}_1^{h_2o} = \dot{E}_2^{h_2o} - \dot{E}_1^{h_2o}$$

$$\dot{E}_1^{h_2o} - \dot{E}_2^{h_2o} = \dot{m}_{h_2o}\left(h_1 + g z_1 + \frac{\vec{V}_1^2}{2}\right) - \dot{m}_{h_2o}\left(h_2 + g z_2 + \frac{\vec{V}_2^2}{2}\right)$$

$$z_1^{h_2o} \cong z_2^{h_2o}, \quad \vec{V}_1^{h_2o} = \vec{V}_2^{h_2o} \Rightarrow \dot{E}_1^{h_2o} - \dot{E}_2^{h_2o} = \dot{m}_{h_2o}(h_1 - h_2)$$



~~$h_1 - h_2 = + (h_{1f} - h_{2f}) - h_{fg}$~~

$h_1 \to h_f$ (ice water, 0°C, 1atm)

$h_2 \to h_g$ (100% steam, 100°C, 1atm)

$$\Rightarrow \dot{E}_1^{h_2o} - \dot{E}_2^{h_2o} = \dot{m}_{h_2o} C_p^{h_2o} (T_1^{h_2o} - T_2^{h_2o}) - \dot{m}_{h_2o} h_{fg}$$

heat water to 100°C

evaporate water into steam

$$\Rightarrow \eta = \frac{\dot{m}_{oil} C_p^{oil} (T_1^{oil} - T_2^{oil})}{\dot{m}_{h_2o} C_p^{h_2o} (T_2^{h_2o} - T_1^{h_2o}) + \dot{m}_{h_2o} h_{fg}} = \frac{\dot{Q}_2^{oil}}{\dot{Q}_1^{h_2o}} = \frac{\dot{E}_1^{oil} - \dot{E}_2^{oil}}{\dot{E}_2^{h_2o} - \dot{E}_1^{h_2o}}$$