A five (5) step cycle does work by changing the temperature, pressure, and volume of a gas. Initially the gas is at P_1, T_1, and V_1. Then the gas is expanded at constant pressure. Then, the gas is expanded at constant temperature. Then, the gas is compressed at constant pressure back to V_2. Then, the gas is cooled at constant volume to a pressure of P_5. Finally, the cycle is completed by returning the system to its initial state via a polytropic process.

(a) What is n for the polytropic process?
(b) What is the work done in ten (10) cycles?
(c) Sketch the process for one (1) cycle on a P-V curve.
(d) Sketch the process for one (1) cycle on a T-S curve.

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a) polytropic $\boxed{PV^n = C} = P_1 V_1^n = P_5 V_5^n \Rightarrow P_1 = P_5\left(\frac{V_5}{V_1}\right)^n$

$$\Rightarrow \log P_1 = \log P_5 + \log\left[\left(\frac{V_5}{V_1}\right)^n\right] \Rightarrow \log\left(\frac{P_1}{P_5}\right) = n\log\left(\frac{V_5}{V_1}\right)$$

$$\Rightarrow \boxed{n = \frac{\log[P_1/P_5]}{\log[V_5/V_1]}}$$

b) $\boxed{W_{net} = 10\, W_{one\ cycle}}$, $\boxed{W_{one\ cycle} = \oint_{cycle} P\,dV = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to5} + W_{5\to1}}$

$$W_{1\to2} = \int_{V_1}^{V_2} P\,dV \quad (\text{constant pressure}) \Rightarrow W_{1\to2} = \int_{V_1}^{V_2} P_1\,dV = P_1 V\Big|_{V_1}^{V_2} = P_1(V_2 - V_1)$$

$$W_{2\to3} = \int_{V_2}^{V_3} P\,dV \quad (\text{constant temp}) \Rightarrow PV = mR_{gas}T = \text{constant}$$

$$= \int_{V_2}^{V_3} \frac{mR_{gas}T}{V}\,dV = mR_{gas}T(\ln V_3 - \ln V_2) = mR_{gas}T_2 \ln\left(\frac{V_3}{V_2}\right) = W_{2\to3}$$

$$W_{3\to4} = \int_{V_3}^{V_4} P\,dV \quad (\text{constant pressure}) \Rightarrow \int_{V_3}^{V_4} P_3\,dV = P_3(V_4 - V_3) = W_{3\to4}$$

$$W_{4\to5} = \int_{V_4}^{V_5} P\,dV \,(\text{constant volume}) \Rightarrow V_5 = V_4 \Rightarrow W_{4\to5} = 0$$

b) cont.

$$\boxed{W_{5\to1} = \int_{V_5}^{V_1} P\,dV} \qquad \boxed{\text{polytropic} \Rightarrow PV^n = \text{constant}}$$

$$\boxed{PV^n = C} = P_1V_1^n = P_5V_5^n \Rightarrow n = \frac{\log[P_1/P_5]}{\log[V_5/V_1]}$$

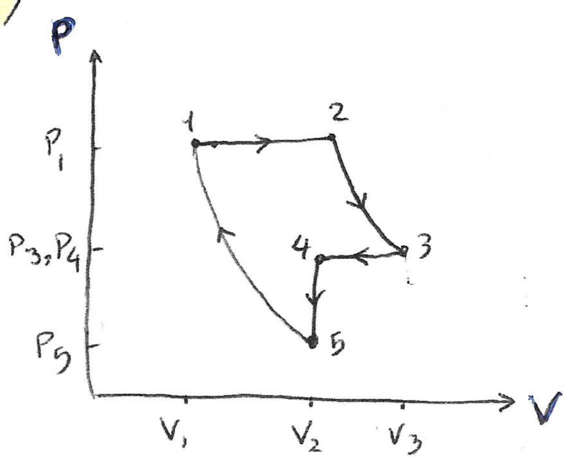
$$\Rightarrow W_{5\to1} = \int_{V_5}^{V_1}[CV^{-n}]\,dV = C\int_{V_5}^{V_1} V^{-n}\,dV = C\frac{V^{-n+1}}{-n+1}\Bigg|_{V_5}^{V_1} = \frac{C}{-n+1}\left[V_1^{-n+1} - V_5^{-n+1}\right]$$

$$W_{5\to1} = \frac{(P_1V_1^n)V_1^{-n+1} - (P_5V_5^n)V_5^{-n+1}}{1-n} \Rightarrow \underline{\frac{P_1V_1^1 - P_5V_5^1}{1-n} = W_{5\to1}}$$

$$\boxed{W_{net} = 10\left[P_1(V_2-V_1) + mR_{gas}T_2\ln(V_3/V_2) + P_3(V_4-V_3) \overset{(-)}{+} \left(\frac{P_1V_1 - P_5V_5}{1-n}\right)\right]}$$

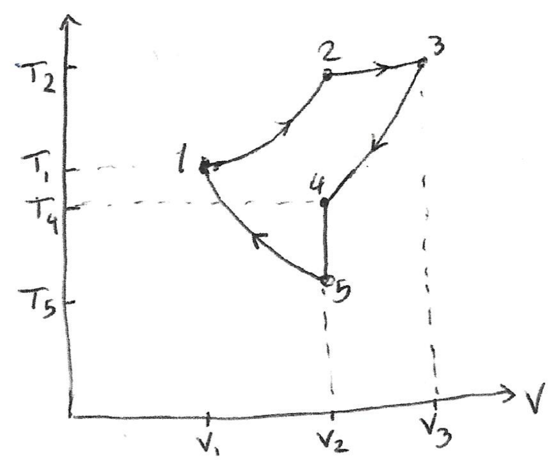
$$n = \log[P_1/P_5]/\log[V_5/V_1]$$

c) P-V



$P_5 < P_4$ ← cooling @ constant volume

d) T-S



$P_4 < P_1 \Rightarrow T_4 < T_1$

$PV \propto T$