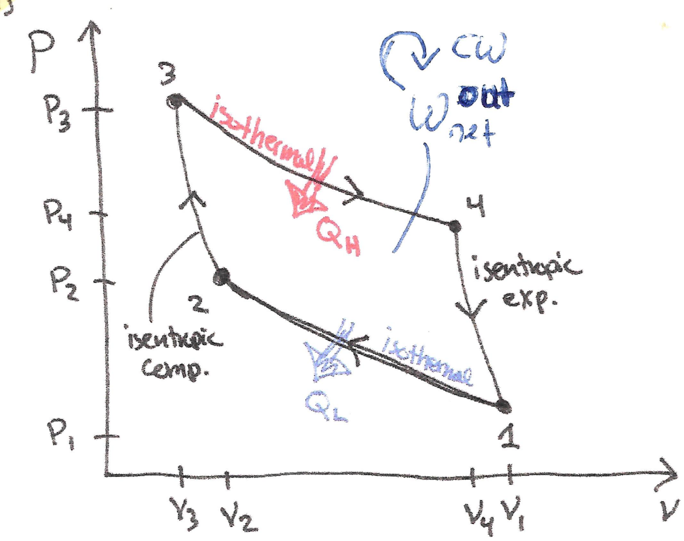
Carnot Cycle: The Carnot cycle is the most efficient power producing cycle that can be executed between a heat source at T_{HOT} and heat sink at T_{COLD}. The cycle is said to be reversible and can be described by the four-step cycle (process):

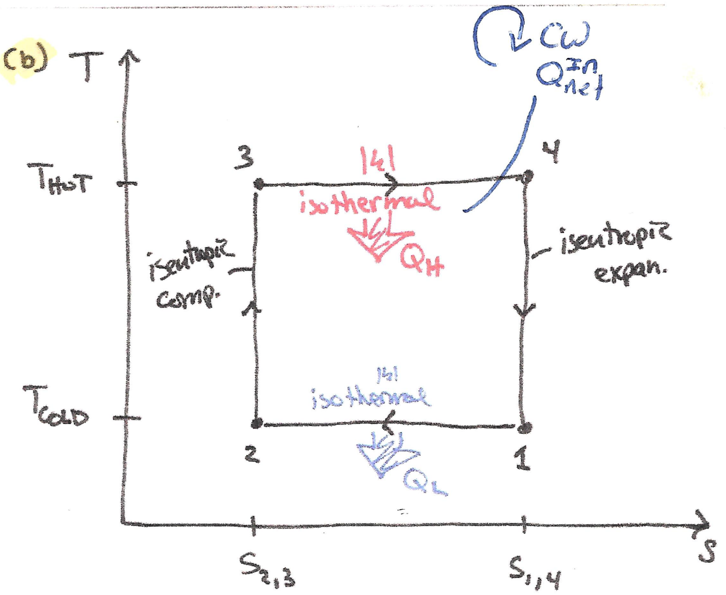
1→2: Isothermal Compression with heat rejection (Q_L) at T_{COLD},
2→3: Isentropic Compression to the temperature of the heat source (T_{HOT}),
3→4: Isothermal Expansion with heat addition (Q_H) at T_{HOT},
4→1: Isentropic Expansion to the temperature of the heat sink (T_{COLD}).

(a) Sketch the P vs. v diagram for the Carnot cycle (label all key parameters and processes). [7 pts]
(b) Sketch the T vs. s diagram for the Carnot cycle (label all key parameters and processes). [7 pts]
(c) Derive the thermal efficiency of the Carnot cycle in terms of T_{COLD} and T_{HOT}? [11 pts]

(a)



(b)



(c) $\boxed{\eta_{th} = \dfrac{W_{net}^{out}}{Q_H} = 1 - \dfrac{Q_L}{Q_H}}$

$[Q_L, Q_H]$ ← derive by/with 1^{st} and/or 2^{nd} laws.

use 2^{nd} law → $TdS = Q$ (Tds generation)

$-Q_L = T_{COLD}(s_2 - s_1)$

$\Rightarrow \boxed{Q_L = T_{COLD}(s_1 - s_2)}$ ✓

$\boxed{+Q_H = T_{HOT}(s_4 - s_3)}$ ✓

isentropic processes...

$\boxed{s_1 = s_4, \quad s_2 = s_3}$

$$\eta_{th} = 1 - \frac{T_{COLD}(s_1 - s_2)}{T_{HOT}(s_4 - s_3)} = 1 - \frac{T_{COLD}(s_4 - s_3)}{T_{HOT}(s_4 - s_3)}$$

$$\Rightarrow \boxed{\eta_{th} = 1 - \frac{T_{COLD}}{T_{HOT}}}$$

Ideal Brayton Cycle: The ideal Brayton cycle represents an ideal, closed cycle for a Gas-Turbine engine. The ideal Brayton cycle is described by the four-step cycle (process):

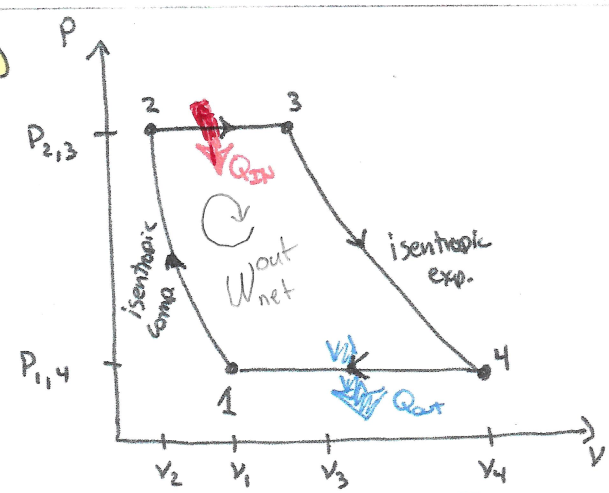
- $T\uparrow$ 1→2: Isentropic Compression (in a **compressor**),
- 2→3: Constant-pressure heat addition (Q_{in}) to v_3,
- 3→4: Isentropic Expansion from v_3 to v_4 (in a **turbine**), W_{in} for compressor?
- 4→1: Constant-pressure compression with heat rejection (Q_{out}) back to v_1.

(a) Sketch the P vs. v diagram for this Brayton cycle (label all key parameters and processes). [7 pts]

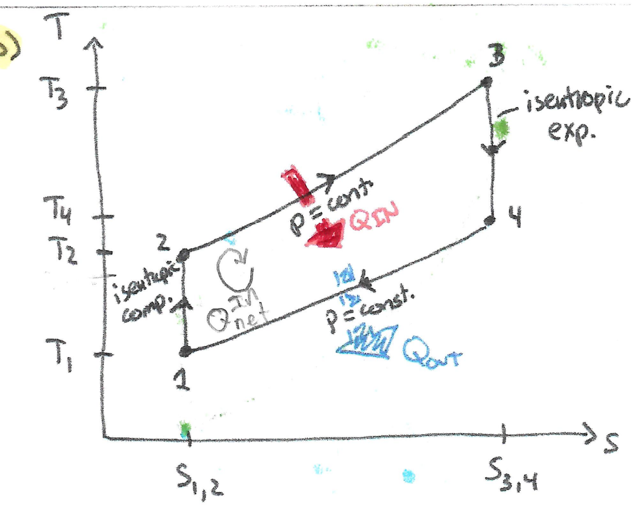
(b) Sketch the T vs. s diagram for this Brayton cycle (label all key parameters and processes). [7 pts]

(c) Derive the thermal efficiency of the ideal Brayton cycle in terms of T_1, T_2, T_3, and T_4? [11 pts]

(a)



(b)



(c) $\boxed{\eta_{th} = \dfrac{W_{net}^{out}}{Q_H} = 1 - \dfrac{Q_{out}}{Q_{IN}}}$

[Q_{out}, Q_{IN}] ← via 1st or 2nd laws

Q_{out}: 1st law, 4→1

$$(\cancel{E_{mass}^{IN}} + \cancel{Q_{IN}} + W_{IN})_{4\to1} - (\cancel{E_{mass}^{out}} + Q_{out} + \cancel{W_{out}})_{4\to1} = \Delta U\big|_{4\to1}$$

$$\Rightarrow W_{IN}\big|_{4\to1} - Q_{out} = U_1 - U_4 \quad \Rightarrow -Q_{out} = U_1 - U_4 - W_{IN}\big|_{4\to1}$$

NOTE: $W_{IN} = -\int P\,dV = -P_1(V_1 - V_4)$ ⇒ $\boxed{W_{IN}\big|_{4\to1} = P_1(V_4 - V_1)}$

$$\Rightarrow Q_{out} = -(U_1 - U_4) + W_{IN}\big|_{4\to1}$$

$$Q_{out} = (U_4 - U_1) + W_{IN}\big|_{4\to1}$$

$$Q_{out} = (U_4 - U_1) + P_1(V_4 - V_1)$$

Prod. Rule

$$\cancel{\Delta H = \Delta U + \Delta(PV)} = \cancel{\Delta U + P\Delta V + V\Delta P} \quad P = const$$

$$\Rightarrow Q_{out} = \Delta H\big|_{1\to4} \Rightarrow \boxed{Q_{out} = mc_p(T_4 - T_1)}$$

Q_{IN}: 1st law, 2→3

$$\Rightarrow Q_{IN}^{2\to3} - W_{OUT}^{2\to3} = \Delta U\big|_{2-3}$$

$$\Rightarrow Q_{IN} = \Delta U\big|_{2-3} + W_{out}\big|_{2-3} \qquad W_{out} = \int P\,dV$$

$$= \Delta H\big|_{2\to3} \Rightarrow \boxed{Q_{IN} = mc_p(T_3 - T_2)}$$

$$\Rightarrow \eta_{th} = 1 - \frac{mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} \Rightarrow \boxed{\eta_{th}^{\text{ideal Brayton}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}}$$

Isentropic – $\Delta S = 0$, undefined line

Otto Cycle: The Otto cycle represents the ideal cycle for Spark-Ignition engines. The ideal Otto cycle is described by the four-step cycle (process):

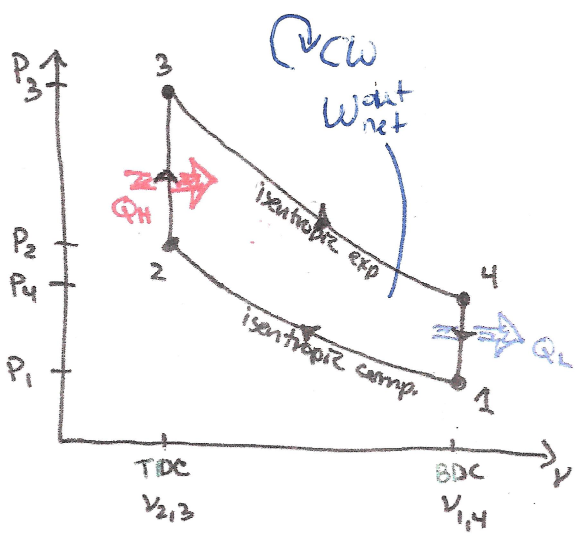
- 1→2: Isentropic Compression from Bottom-Dead-Center (BDC) to Top-Dead-Center (TDC),
- 2→3: Constant volume heat addition (Q_H),
- 3→4: Isentropic Expansion from TDC (V_{min}) to BDC (V_{max}),
- 4→1: Constant volume heat rejection (Q_L).

(a) Sketch the P vs. v diagram for the Otto cycle (label all key parameters and processes). [7 pts]

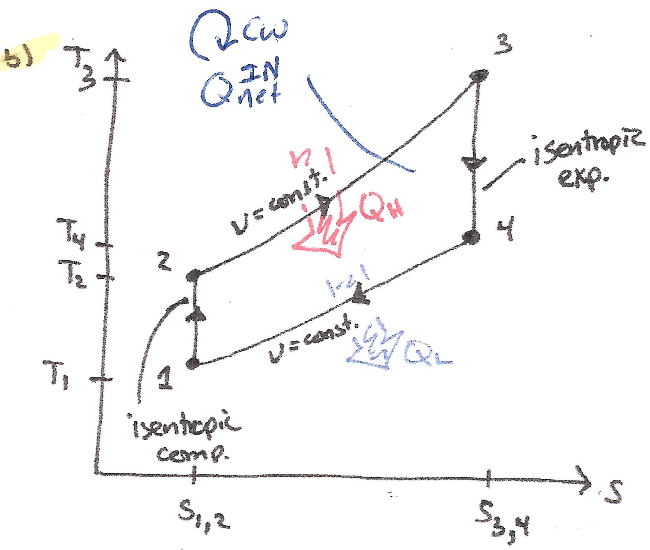
(b) Sketch the T vs. s diagram for the Otto cycle (label all key parameters and processes). [7 pts]

(c) Derive the thermal efficiency of the Otto cycle in terms of T_1, T_2, T_3, and T_4? [11 pts]

(a)



(b)



(c) $\eta_{th}^{otto} = \dfrac{W_{net}^{out}}{Q_H} = 1 - \dfrac{Q_L}{Q_H}$ $[Q_L, Q_H] \leftarrow$ derive via 1st or 2nd laws

via 1st law

Q_L: $\rightarrow$ $\boxed{E_{IN}\big|_{4\to1} - E_{out}\big|_{4\to1} = \Delta U\big|_{4\to1}}$

$$\Rightarrow \left(\cancel{E_{mass,IN}^{4\to1}} + \cancel{Q_{IN}^{4\to1}} + \cancel{W_{IN}^{4\to1}}\right) - \left(\cancel{E_{mass,out}^{4\to1}} + Q_{OUT}^{4\to1} + \cancel{W_{OUT}^{4\to1}}\right) = U_1 - U_4$$

note: $W_{IN}^{4\to1} = -\int_{V_4}^{V_1} P\,dV = 0$

$$\Rightarrow -Q_{out}^{4\to1} = U_1 - U_4$$

$Q_{out}^{4\to1} = Q_L$

$\left.\dfrac{\partial u}{\partial T}\right|_v = c_v$ $\Rightarrow$ $\boxed{Q_L = mc_v(T_4 - T_1)}$ ✓

via 1st law

Q_H: $\boxed{E_{IN}\big|_{2\to3} - E_{out}\big|_{2\to3} = \Delta U\big|_{2\to3}}$

$$\Rightarrow \left[\cancel{E_{mass}^{in}} + Q_{in} + \cancel{W_{in}}\right] - \left[\cancel{E_{mass}^{out}} + \cancel{Q_{out}} + \cancel{W_{out}}\right] = \Delta U = U_3 - U_2$$

IN: 2→3 OUT: 2→3

note: 2→3 (constant volume)

$\Rightarrow W_{IN} = W_{OUT} = 0$

$$\Rightarrow Q_H = U_3 - U_2$$

$$\boxed{Q_H = mc_v(T_3 - T_2)}$$ ✓

$Q_{IN} = Q_H$

$\left.\dfrac{\partial u}{\partial T}\right|_v = c_v$

$$\Rightarrow \eta_{th}^{otto} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

$$\Rightarrow \boxed{\eta_{th}^{otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}}$$

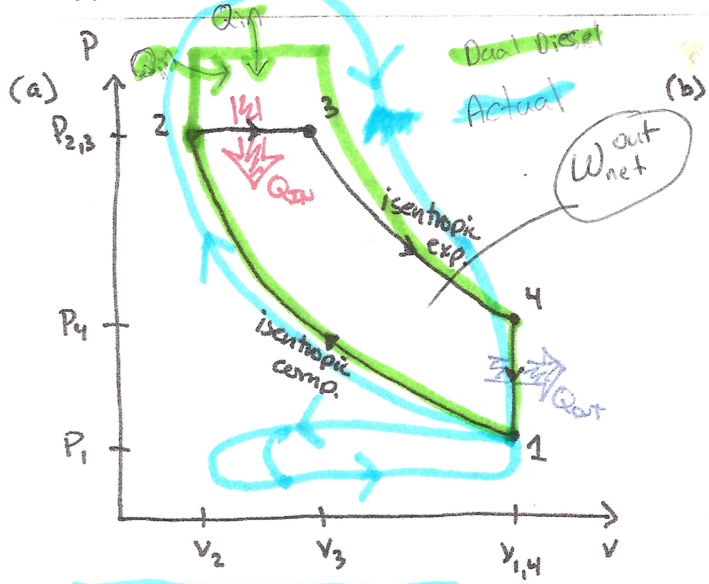
Diesel Cycle: The Diesel cycle represents the ideal cycle for Compression-Ignition engines. The ideal Diesel cycle is described by the four-step cycle (process):

1→2: Isentropic Compression from Bottom-Dead-Center (BDC) to Top-Dead-Center (TDC),
2→3: Constant pressure heat addition (Q_{in}) to v_3,
3→4: Isentropic Expansion from v_3 to v_4,
4→1: Constant volume heat rejection (Q_{out}) at TDC (v_1).

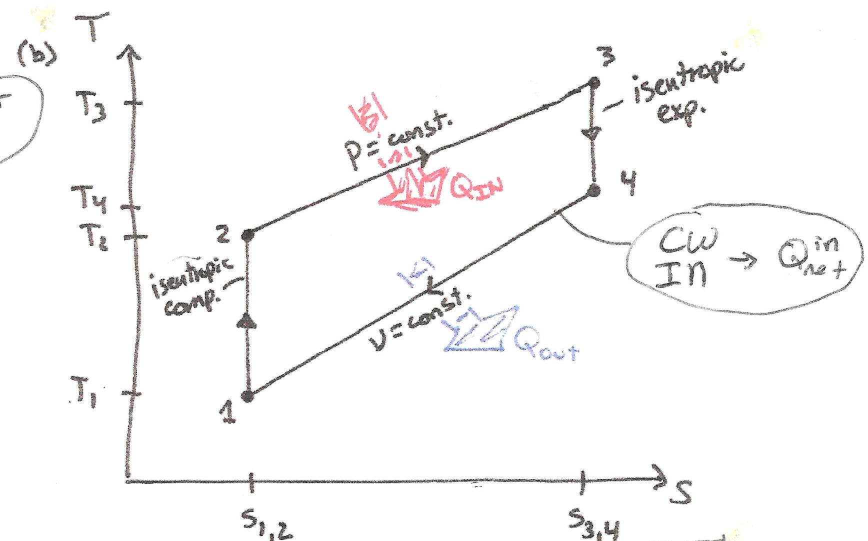
$\Delta S = 0$ for isentropic
$T\uparrow$ for isentropic comp

(a) Sketch the P vs. v diagram for the Diesel cycle (label all key parameters and processes). [7 pts]
(b) Sketch the T vs. s diagram for the Diesel cycle (label all key parameters and processes). [7 pts]
(c) Derive the thermal efficiency of the Diesel cycle in terms of k, T_1, T_2, T_3, and T_4? [11 pts]

(a)



(b)



$K = \frac{C_p}{C_v}$

(c) $\boxed{\eta_{th} = \frac{W_{net}^{out}}{Q_H} = 1 - \frac{Q_L}{Q_H}}$ $[Q_L, Q_H] \leftarrow$ via 1^{st} or 2^{nd} law

$Q_L : (1^{st}\text{ law})\ \xrightarrow[4\to1]{}$ $\left(\cancel{E_{mass}^{IN}} + \cancel{Q_{IN}} + \cancel{W_{IN}}\right)_{4\to1} - \left(\cancel{E_{mass}^{out}} + Q_{out} + \cancel{W_{out}}\right)_{4\to1} = \Delta u\Big|_{4\to1}$

$\Rightarrow -Q_{out} = u_1 - u_4$, $Q_{out} = Q_L$, $\frac{\partial u}{\partial T}\Big|_v = c_v$ $\Rightarrow -Q_L = mc_v(T_1 - T_4)$

$\Rightarrow \boxed{Q_L = mc_v(T_4 - T_1)}$

$Q_H : (1^{st}\text{ law})\ \xrightarrow[2\to3]{}$ $\left(\cancel{E_{mass}^{IN}} + Q_{IN} + \cancel{W_{IN}}\right)_{2\to3} - \left(\cancel{E_{mass}^{out}} + \cancel{Q_{out}} + W_{out}\right)_{2\to3} = \Delta u\Big|_{2\to3}$

$Q_{IN} = Q_H \Rightarrow \boxed{Q_H - W_{out} = u_3 - u_2}$ $\Rightarrow Q_H = \Delta u\Big|_{2\to3} + W_{out}\Big|_{2\to3}$

NOTE: $\Delta H\Big|_{2\to3} = \Delta u\Big|_{2\to3} + \Delta(PV)\Big|_{2\to3} = (u_3 - u_2) + P\Delta V\Big|_{2\text{-}3} + V(\cancel{\Delta P})^{0}\Big|_{2\to3}$ $W_{out}\Big|_{2\to3} = \int_{V_2}^{V_3} PdV = P_2(V_3 - V_2)$

$\Rightarrow Q_H = \Delta H\Big|_{2\to3} \Rightarrow \boxed{Q_H = mc_p(T_3 - T_2)}$

$\Rightarrow \eta_{th} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$ $k = \frac{c_p}{c_v}$

$\Rightarrow \boxed{\eta_{th} = 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)}}$