Ch. 4 Review

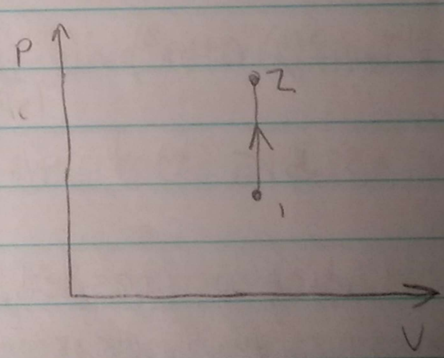
boundary work - associated with an expanding/contracting boundary (piston-cylinders)

$$W_b = \int_1^2 P\,d\forall \quad (kJ)$$

Example) A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done in the process.

$$W = \int P\,d\forall$$

- The tank is rigid, so $d\forall = 0$, therefore $W = 0$.



$U = mC\Delta T$

$$W = \int_1^2 P\,d\forall = P\forall\Big|_1^2 = P\forall_2 - P\forall_1$$

Example) A piston-cylinder device initially contains $0.4m^3$ of air at 100kPa and 80°C. The air is now compressed to $0.1m^3$ in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

- using IG EOS $\qquad 80°C = 353.15K$

$$P_1V_1 = RT_1$$

- In this case, mass, R, and temperature are constant.
- Therefore, $P_1V_1 = mRT = C \qquad (C = \text{a constant})$

$$P = \frac{C}{V}, \text{ since } P \text{ is needed for boundary work.}$$

$$W = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV = C\ln V\Big|_1^2 = C(\ln V_2 - \ln V_1) = C\ln\frac{V_2}{V_1}$$

now, $W = P_1V_1 \ln\frac{V_2}{V_1} = (100\times10^3 \frac{N}{m^2})(0.4m^3)\ln\left(\frac{0.1m^3}{0.4m^3}\right)$

$= -55451.77\ J = -55.5kJ$

- Since the system is being compressed, work is being done on it. Therefore, the work is negative.

In general,

$W = \int_1^2 P\,dV$

Isobaric processes: $W = p(V_2 - V_1)$

Polytropic processes: $W = \frac{P_2 V_2 - P_1 V_1}{1-n}$; $n \neq 1$ ($PV^n = \text{constant}$)

Isothermal IG: $W = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_0 \ln \frac{V_2}{V_1}$ ($PV = mRT_0 = \text{const}$)

4-8) $m = 0.6\,kg$

State 1		State 2
$P_1 = 1.0\,MPa$	$V_2 = .4V_1$	$P_2 = 1.0\,MPa$
$T_1 = 400°c$		$T_2 = 250°c$

I) -State 1 is superheated. (At 1 MPa, T=179.88 on table) (Also, 400°c is too high for saturated.)

II) -Using superheated tables, at 1.0 MPa, $\underline{v_1 = .30661 \frac{m^3}{kg}}$

III) -State 2 is superheated. (At 1 MPa, T=179.88 on table. Also, at 250°c, pressure is 3976.2 kPa; much higher than our 1 MPa (1000 kPa).)
-Don't assume state 2 is superheated!! There is a large temperature drop and it could have cooled enough to change phase.

IV) At state 2, $\underline{V_2 = .23275 \frac{m^3}{kg}}$.

Using the boundary work equation,

$$W = \int_1^2 p \, d\forall$$

- This is a constant-pressure process. Therefore,

$$W = p(\forall_2 - \forall_1)$$

- We use the two specific volumes found, and we know the mass of steam. This system is closed, so no mass exits or enters.

$$W = p[m(v_2 - v_1)] \qquad d\forall = m(v_2 - v_1) \text{ or } \forall_2 - \forall_1 \qquad v_2 = .23275 \frac{m^3}{kg}$$

$$v_1 = .30661 \frac{m^3}{kg}$$

$(1\,Pa = 1 \frac{N}{m^2})$ $$W = 1.0 \times 10^6 \frac{N}{\cancel{m^2}} \left[0.6\,\cancel{kg} (.23275 - .30661) \frac{m^{\cancel{3}}}{\cancel{kg}}\right]$$

$(1\,J = 1\,N \cdot m)$ $$W = -44316\ J = \boxed{-44.316\ kJ}$$

b) State1

$P_1 = 1.0\,mPa$

$T = 400^\circ C$

$\forall_2 = .4\forall_1$

$m = .6\,kg$

State2

$P_2 = 0.5\,mPa$

$T_2 = ?$

-From part a, $V_1 = .30661 \frac{m^3}{kg}$

-Using the given volume relation, we can find the new V_2.

-In this problem, work is not being done all the way until the end. There is an intermediate state when the piston first hits the stops. This is where you need to end the boundary work calculation.

-All we know about this intermediate state is that the pressure remains constant and that the volume is 40% of the initial volume.

-Knowing this, $\forall_{stop} = .4\forall_1$, and $V_{stop} = .4V_1$

$V_{stop} = .4(.30661 \frac{m^3}{kg}) = .12264 \frac{m^3}{kg}$

Now, $W_{1 \to stop} = P_1 m (V_{stop} - V_1) = (1.0\,mPa)(.12264 - .30661)\frac{m^3}{kg}(0.6\,kg)$

$W_{1 \to stop} = -110.3\,KJ$

-This process is different since we cannot use property tables for V_{stop}.

c) $T_2 = ?$

P_2 and V_2 are known.

-Using tables (A-4), $T_2 = 151.83^\circ C$.

Ex.5-4) Air at 10°c and 80 kPa enters a diffuser of a jet engine st... with a velocity of $200 \frac{m}{s}$. The inlet area is $0.4 m^2$. The air leaves at a velocity that is very small compared with the inlet velocity. Determine a) the mass flow rate of the air and b) the temperature of the air leaving the diffuser.

a) State 1

$T_1 = 10°C = 283.15 K$

$P_1 = 80 kPa$

$V_1 = 200 \frac{m}{s}$

State 2

$T = ?$

$V_2 \cong 0$

-Using IG formula, specific volume is needed to obtain the mass flow rate.

$$Pv = RT \rightarrow v = \frac{RT}{P} = \frac{\left(287 \frac{\cancel{N} \cdot m}{kg \cdot \cancel{K}}\right)(283.15 \cancel{K})}{80 \times 10^3 \frac{\cancel{N}}{m^2}} = 1.0158 \frac{m^3}{kg}$$

$\dot{m} = \frac{1}{v} VA$ (since v is the reciprocal of density)

$$\dot{m} = \left(\frac{1}{1.0158 \frac{m^3}{kg}}\right)\left(200 \frac{m}{s}\right)\left(0.4 m^2\right) = 78.75 \frac{kg}{s}$$

-Create an expression for h_2 using $\dot{E}_{in} = \dot{E}_{out}$

$$\cancel{\dot{m}}\left(h_1 + \frac{V_1^2}{2}\right) = \cancel{\dot{m}}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$h_2 = h_1 + \frac{V_1^2 - V_2^2}{2}$$ (since $V_2 \approx 0$, cancel it)

$$h_2 = h_1 + \frac{V_1^2}{2}$$

-From IG tables, $h_1 = 283.14 \frac{kJ}{kg}$ (at 283K) (interpolated between 280 and 285 using h and T)

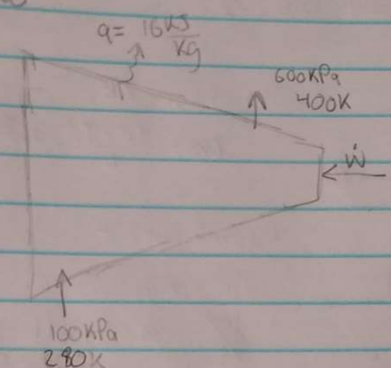
$h_2 = 303.14 \frac{kJ}{kg}$ Interpolating: $T_2 \approx 303K$

6-13-19

(Seen on GroupMe)

Air at 100 kPa, 280 K is compressed to 600 kPa, 400 K. $\dot{m} = 0.02\,\frac{kg}{s}$, heat loss of $16\,\frac{kJ}{kg}$. Determine work input.

$\Delta KE, PE = 0$



$$\dot{Q} = q\dot{m} = \left(16\,\frac{kJ}{kg}\right)\left(0.02\,\frac{kg}{s}\right)$$

$$\dot{Q} = -.32\,kJ$$

$$\cancel{\frac{dE_{cv}}{dt}} = \dot{Q} - \dot{W} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$\dot{W} = Q + \dot{m}(h_1 - h_2)$ (Rearrange for work)

-Using tables, $h_1 = 280.13\,\frac{kJ}{kg}$; $h_2 = 400.98\,\frac{kJ}{kg}$

$$\dot{W} = -.32\,kJ + \left(.02\,\frac{kg}{s}\right)(280.13 - 400.98)\,\frac{kJ}{kg}$$

$\dot{W} = -2.737\,kW$

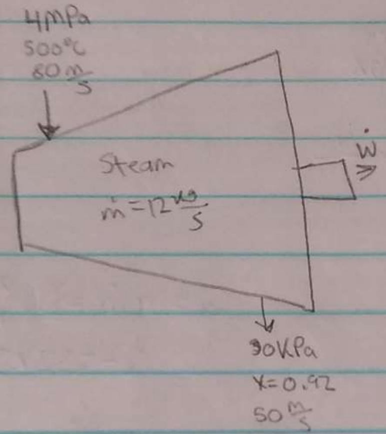
(negative since it is work input)

or

$\dot{W}_{in} = 2.737\,kW$

For these problems, I was taught to use the general energy balance equation (shown below the drawing), so that's what I was used to. Using the Ein=Eout method will yield the same results. The only difference between the two methods is that I have to account for signs using the general energy balance equation.

5-48) Steam flows steadily through an adiabatic turbine. The inlet conditions of the steam are 4MPa, 500°C, and 80 $\frac{m}{s}$, and the exit conditions are 30kPa, .92, ... and 50 $\frac{m}{s}$. The mass flow rate is 12 $\frac{kg}{s}$.



(using $\frac{1kJ}{kg} = 1000\frac{m^2}{s^2}$)

a) change in KE (where V = velocity)

$$\Delta KE = \frac{V_2^2 - V_1^2}{2} = \frac{50^2 - 80^2}{2} = -1950\frac{m^2}{s^2} = -1.95\frac{kJ}{kg}$$

b) power output

-use energy balance

$$\frac{dE_{cv}}{dt} = \cancel{\dot{Q}} - \dot{W} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\dot{W} = \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

- Gathering h values

$h_1 = 3446\frac{kJ}{kg}$ (superheated table) ; $h_2 = h_f + x(h_{fg}) = 289.27 + .92(2335.3)$

$h_2 = 2437.746\frac{kJ}{kg}$ (sat. table)

$$\dot{W} = \left(12\frac{kg}{s}\right)\left[(3446 - 2437.746)\frac{kJ}{kg} + \frac{(80\frac{m}{s})^2 - (50\frac{m}{s})^2}{2}\left(\frac{1\frac{kJ}{kg}}{1000\frac{m^2}{s^2}}\right)\right]$$

$\dot{W} = 12122\,kW = 12.122\,MW$

c) turbine inlet area

$$\frac{\left(\frac{kg}{s}\right)\left(\frac{m^3}{kg}\right)}{\frac{m}{s}}$$ m^2

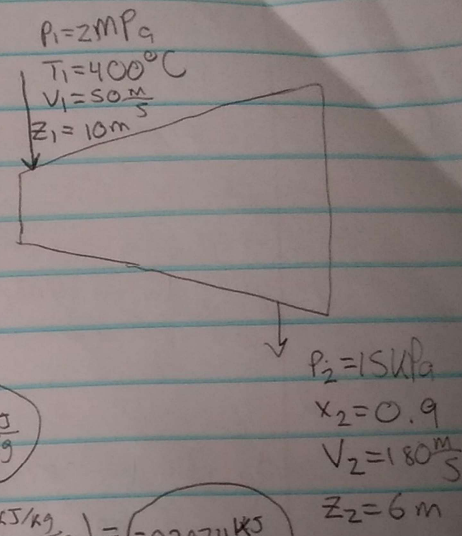
$V_1 = .08644\frac{m^3}{kg}$

$\dot{m} = \frac{1}{v_1}VA$

$A = \frac{\dot{m}v_1}{V_1}$

$$A = \frac{\left(12\frac{kg}{s}\right)\left(.08644\frac{m^3}{kg}\right)}{80\frac{m}{s}} = 0.0130\,m^2$$

Ex 5-7) Power output = 5MW (Turbine)



a) compare magnitudes of $\Delta h, \Delta ke, \Delta pe$

$h_1 = 3248.4 \frac{KJ}{kg}$ (superheated tables)

$h_2 = h_f + x(h_{fg}) = 225.94 + 0.9(2372.3)$

$h_2 = 2360.99$

$\Delta h = h_2 - h_1 = 2360.49 - 3248.4 = -887.41 \frac{KJ}{Kg}$

$\Delta pe = g(z_2 - z_1) = 9.81(6-10) = -39.24 \frac{m^2}{s^2}\left(\frac{1KJ/kg}{1000\frac{m^2}{s^2}}\right) = .03924\frac{KJ}{kg}$

$\Delta ke = \frac{v_2^2 - v_1^2}{2} = \frac{180^2 - 50^2}{2} = 14950 \frac{m^2}{s^2} = 14.95\frac{KJ}{Kg}$

b) work per unit mass

$\dot{E}_{in} = \dot{E}_{out}$

$\cancel{\dot{m}}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \cancel{\dot{m}}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \frac{\dot{W}_{out}}{\dot{m}}$

$\frac{\dot{W}_{out}}{\dot{m}} = -\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$

$= -(-887.41 + 14.95 - .03924) = 872.49 \frac{KJ}{kg}$

c) $\dot{m}$

$\dot{m} = \frac{\dot{W}_{out}}{\frac{W_{out}}{\dot{m}}} = \frac{5000\frac{KJ}{s}}{872.49\frac{KJ}{kg}} = 5.73 \frac{Kg}{s}$

On this problem, I started using Ein=Eout to avoid confusion during tutoring.

Ch.6

Second Law

- Doesn't involve energy conservation.
- For example, a cup of hot coffee getting hotter due to the air in the room doesn't violate the first law, but it violates the second law.
- Second law concerns "reversibility"

Heat Engines

- Operate in a cycle
- Receive heat from a high-temp. reservoir and convert some amount of heat to work.

Thermal Efficiency

$$\eta_{th} = \frac{\text{Net work output}}{\text{heat input}} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \qquad W_{cyc} = Q_{cyc}$$

$Q_H = Q_{in}$ = heat transfer at high-temp. medium (T_H)

$Q_L = Q_{out}$ = heat transfer at low-temp. medium (T_L)

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

6.6-1) Heat is transferred to a heat engine from a furnace at a rate of 80mW. If the rate of waste heat rejection to a nearby river is 50mW, determine the net power output and the thermal efficiency of the engine.

$Q_{in} = 80mW \quad Q_{out} = 50mW$

$W_{cyc} = Q_{cyc} \rightarrow 80 - 50 = 30mW$

$\eta_{th} = \frac{W_{cyc}}{Q_{in}} = \frac{30mW}{80mW} = \frac{3}{8} = .375 = 37.5\%$

Kelvin-Planck Statement

It is impossible for any device that operates cyclically to receive heat from a single source and convert that heat completely into work.

- There has to be some amount of waste heat.

Refrigeration Cycles + Heat Pumps

-Coefficient of performance (COP)

Refrigeration

$COP_R = \frac{Q_L}{W_{in}}$ (the objective of a refrigerator is to reject heat)

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Heat Pumps

$$COP_{HP} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

(The objective of a heat pump is to absorb heat from a cold source and move it to a hot reservoir)

Ex 6-3) A refrigerator with COP of 1.2 removes heat from a refrigerated space at a rate of 60 $\frac{kJ}{min}$. Determine a) the electric power consumed by the refrigerator and b) the rate of heat transfer to the kitchen air.

$COP = \frac{Q_L}{W_{in}}$ $\qquad Q_L = 60\frac{kJ}{min}\left(\frac{1 min}{60 s}\right) = 1 KW$

a) $W_{in} = \frac{Q_L}{COP} = \frac{60\frac{kJ}{min}}{1.2} = 50\frac{kJ}{min}\left(\frac{1 min}{60 s}\right) = .83 KW$

b) $Q_{cyc} = W_{cyc}$

$W_{in} = Q_H - Q_L$

$50 = Q_H - 60$

$Q_H = 110\frac{kJ}{min}$

Q_H = heat loss

Q_L = heat absorbed

6-4) HP is used to meet the heating requirements of a house and maintain it at 20°C. When outdoor temp. drops to -2°C, the house loses heat at 80000 $\frac{kJ}{h}$. The COP = 2.5

Determine:

a) power consumed, b) rate at which heat is absorbed from outside.

a) $COP = \frac{Q_H}{W_{in}}$; $W_{in} = \frac{Q_H}{COP} = \frac{80000 \frac{kJ}{h}}{2.5} = 32000 \frac{kJ}{h} \left(\frac{1 hr}{3600 s}\right) = 8.88 kW$

- $\dot{Q}_H$ is 80000 since that heat must be kept constant to maintain temperature.

$$HP \xrightarrow{\dot{Q}_H} \boxed{House} \rightarrow 80000 \frac{kJ}{h}$$

b) $COP = \frac{Q_H}{Q_H - Q_L}$

$$Q_L = -\frac{Q_H}{COP} + Q_H = \frac{-80000}{2.5} + 80000 = 48000 \frac{kJ}{h}$$

HP = Heat Pump. Also, Q_H and Q_L signify different things depending on the cycle being analyzed. For power cycles, Q_H is the heat that goes *into* the cycle, since it is what powers the cycle. Q_L is any sort of heat loss in a power cycle.

For refrigeration cycles and heat pumps, Q_H is actually the heat *lost* and Q_L is the heat *absorbed*. This is because heat pumps and refrigerators can be thought of as the same cycle but measured from different standpoints.

A refrigerator's objective is to cool something by removing heat from whatever is being cooled. That heat must be rejected somewhere, which is a hot reservoir. Therefore, the COP of a refrigerator is measured by comparing the heat taken out of the cooled area (heat into the refrigeration cycle) as compared to the power input to the refrigerator. For heat pumps, the goal is to warm the hot reservoir even further by removing heat from a cold reservoir. This is why the COP for a heat pump is found by comparing how much warmer the hot reservoir is (heat rejected by the cycle into the hot reservoir) and the power input to the cycle.

Carnot heat engine

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} \quad \text{(Carnot efficiency)}$$

$\eta_{th} < \eta_{th,rev}$ = irreversible

$\eta_{th} = \eta_{th,rev}$ = reversible

$\eta_{th} > \eta_{th,rev}$ = impossible

6-5) A Carnot heat engine receives 500KJ of heat per cycle from a high-temp. source at 652°C and rejects heat to a low-temp. sink at 30°C. Determine a) η_{th} and b) amount of heat rejected to the sink.

a) $$\eta_{th\,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{30+273}{652+273} = .672$$

b) Q_L

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} \qquad Q_L = -Q_H(\eta_{th} - 1) = -500KJ(.672-1) = 164KJ$$

Carnot Refrig. + HP

$$COP_{R,rev} = \frac{1}{\frac{T_H}{T_L} - 1} \qquad COP_{HP,rev} = \frac{1}{1 - \frac{T_L}{T_H}}$$

$COP_R < COP_{R\,rev}$ = irreversible

$COP_R = COP_{R\,rev}$ = reversible

$COP_R > COP_{R\,rev}$ = impossible

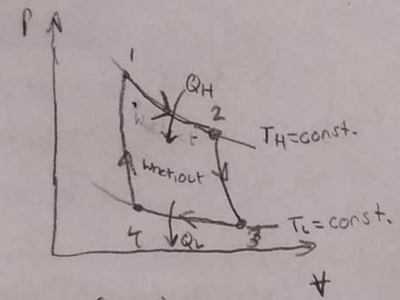
$$COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad COP_{HP} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Carnot Cycle

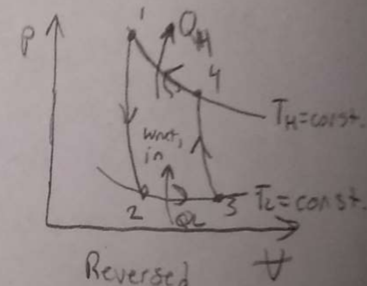
- Reversible cycle (ideal)

Carnot cycle steps: Isothermal expansion → Adiabatic expansion → Isothermal c[illegible]
↓
Adiabatic comp.

Isothermal – Adiabatic – Isothermal – Adiabatic



Carnot
Heat engine



Reversed
Carnot
(Carnot refrigeration cycle)

- The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same reservoirs
  (Reversible cycles are more efficient)

- Efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{T_H}{T_L} \text{ (absolute)}$$

6-7) A heat pump is to be used to heat a house. The house is to be maintained at 21°C. It is losing heat at 135000 $\frac{kJ}{h}$ when outside temp. drops to -5°C. Determine the minimum power required for the pump.

$$\dot{Q}_H = 135000 \frac{kJ}{h}$$

$T_L = 268K$

$T_H = 294K$

$$COP = \frac{1}{1-\frac{T_L}{T_H}} = \frac{1}{1-\frac{268}{294}} = 11.308$$

$$COP = \frac{\dot{Q}_H}{W_{in}} \qquad W_{in} = \frac{\dot{Q}_H}{COP} = \frac{135000\frac{kJ}{h}}{11.308} = 11938.45\frac{kJ}{hr}\left(\frac{1hr}{3600s}\right)$$

$= 3.32 kW$

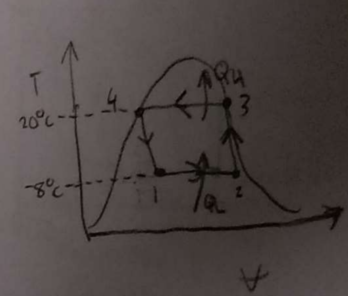
In the book, this is example 6-7. This problem is assumed to be reversible since it is in the Carnot devices section. Usually, there will be more information on whether or not the cycle is Carnot. It will use words such as “reversible”, “ideal”, etc., or may even outright tell you it is a Carnot device.

6-6) A Carnot refrig. cycle is executed in a closed system in the saturated liquid-vapor region using 0.8 kg of R-134a. $T_H = 20°C$, $T_L = -8°C$. The refrigerant is a sat. liq. at the end of the heat rejection process and the net work input is 15 kJ. Determine the fraction of the mass of refrigerant that vaporizes during the heat addition process and the pressure at the end of the heat rejection process.

$T_H = 293K$

$T_L = 265K$

$$COP_R = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\frac{293}{265} - 1} = 9.46$$



heat addition: 1-2

State 1

$T_1 = -8°C$

State 2

$T_2 = -8°C$

vaporization) $Q_L = m_{evap} h_{fg}$

Using HP equation, $COP = \frac{Q_L}{W_{in}}$

$$m_{evap} = \frac{Q_L}{h_{fg}} = \frac{141.9 kJ}{204.59 \frac{kJ}{kg}} = 0.694 kg$$

$$Q_L = COP\, W_{in} = 9.46(15 kJ) = 141.9 kJ$$

$$\text{Fraction of } m_{evap} = \frac{0.694}{0.8} = .868 = 86.8\%$$

b) pressure at end of heat rejection process (state 4)

at state 4, $T = 20°C$ and it is sat. liq.

$P_4 = 572.1 kPa$

This is Example 6-6 from within the text.

GOOD LUCK ON YOUR EXAM!